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An approximate analytical model for footprint estimation of scalar fluxes in thermally stratified atmospheric flows

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Abstract

An approximate analytical model was developed to estimate scalar flux footprint in thermally stratified atmospheric surface layer flows. The proposed model was based on a combination of Lagrangian stochastic dispersion model results and dimensional analysis. The main advantage of this model is its ability to analytically relate atmospheric stability, measurement height, and surface roughness length to flux and footprint. Flux estimation by the proposed model was in good agreement with those calculated by detailed Eulerian and Lagrangian models. Measured water vapor fluxes collected along a downwind transect of a transition from a desert to an irrigated potato site were also used to assess the proposed model performance in the field. It was found that the model well reproduced the measured flux evolution with downwind distance. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Scalar flux footprint estimation continues to be a practical research problem in surface hydrology and boundary layer meteorology [4,11,14,23,27]. Such an analysis is commonly used to quantify the contributing source areas to scalar flux measurement or to examine adequate fetch requirements. Both Eulerian analytical models [6,8] and Lagrangian stochastic dispersion models [5,12,16] with varying degrees of complexity have been used to investigate the relationship between scalar flux and its source areas.

For example (as early as 1986), Gash [6] developed a simple model for footprint calculation for neutral atmospheric conditions. Later, Horst and Weil [7,8] proposed an Eulerian analytical model that is capable of incorporating atmospheric stability, albeit the model treatment of atmospheric stability effects on fetch is not explicit. Leclerc and Thurtell [16] first applied a Lagrangian particle trajectory model to examine "rule of thumb" fetch requirement and found that the "100 to 1 fetch-to-height ratio grossly underestimates fetch requirements when observations are carried out above smooth surfaces, in stable conditions, or at high observation level". These calculations highlight the important roles of measurement height, surface roughness, and atmospheric stability on footprint estimation. Finn et al. [4] examined the performance of Eulerian [7,8] and Lagrangian [16] models for estimating footprint. They concluded that while Eulerian models are easier to implement, they should be used with caution over rough terrain. In short, despite all such advancements, none of the existing models explicitly describes the relationship between footprint, atmospheric stability, observation height, and surface roughness.

The objective of this study is to develop an approximate analytic expression, analogous to Gash [6], that accurately describes the relationship between footprint, observation height, surface roughness, and atmospheric stability. For this purpose, similarity theory (dimensional analysis) in conjunction with the Lagrangian stochastic dispersion model [25] simulation outputs are used to construct the relationships among parameters (i.e., flux, fetch, atmospheric stability, surface roughness, and observation height) and to develop the model framework. Comparisons with existing Lagrangian and Eulerian models are also performed. The model is also field-tested using water vapor flux measurements

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$$a = -\frac{(w-W)}{t_L} + \frac{1}{2}\frac{\partial\sigma_w^2}{\partial z}\left(1+\frac{(w-W)^2}{\sigma_w^2}\right) + \frac{1}{2}\frac{\partial\sigma_w^2}{\partial z}\left(\frac{W(w-W)}{\sigma_w^2}\right) + \frac{\partial W}{\partial z}(W+(w-W)), \quad (11)$$

$$b = \left(\frac{2\sigma_w^2}{t_L}\right)^{1/2}, \quad (12)$$

where W, the mean Eulerian vertical velocity, is zero in the atmospheric surface layer, σ_w^2 the Eulerian vertical velocity variance, and t_L is the Lagrangian decorrelation time scale. By releasing a large number of particles and using (9a)–(12), and upon specifying the Eulerian velocity statistics profiles and Lagrangian time scale (see Appendix B for details), particle trajectories can be computed, and subsequently, the scalar fluxes and footprint. The scalar flux, F, at the point (x, z_m) is calculated as

$$F(x, z_m) = \frac{S_0}{N}(n_\uparrow - n_\downarrow), \quad (13)$$

where $n_\uparrow$ and $n_\downarrow$ are the number of particles which reach the height z_m at position x with upward and downward vertical velocity, respectively, and N is the total number of particles released. The footprint, f, is then calculated from

$$f(x, z_m) = \frac{1}{S_0}\frac{dF(z, z_m)}{dx}.$$

2.4. Proposed model

From previous model results [8,12,18,22], the fetch (x) is a function of F, z_m, z_0, and the atmospheric stability parameter (z_m/L), where L is the Obukhov length (see Appendix A for definition). From (3) and (4) z_m and z_0 can be combined to form a new length scale, z_u, defined as

$$z_u = z_m(\ln(z_m/z_0) - 1 + z_0/z_m).$$

Hence, we have three characteristic length scales: x, L, and z_u for the dimensional analysis. With L as the key variable, we propose the following two dimensionless groups (Pi groups [20,24]) and write

$$\frac{x}{L} = f\left(\frac{z_u}{L}\right). \quad (14)$$

Using Thomson's [25] Lagrangian model mentioned above, we calculated the 90% flux fetch requirements (i.e., the x values for reaching $F/S_0 = 0.9$) for a wide range of z_m, z_0, and L values. The 90% flux fetch requirement is the needed downwind distance, x (from the transition), for the measured flux, $F(x, z_m)$, to present 90% of the surface flux, S_0. In our calculations, z_m ranges from 2 to 20 m; z_0 ranges from 0.01 to 0.1 m; L ranges from −0.1 to 50 m. Fig. 1(a)–(c) shows how $x/|L|$ varies with $z_u/|L|$ for unstable, neutral, and stable atmospheric conditions, respectively. From Fig. 1(a)–(c) and in accordance with (3), we can express $x/|L|$ as

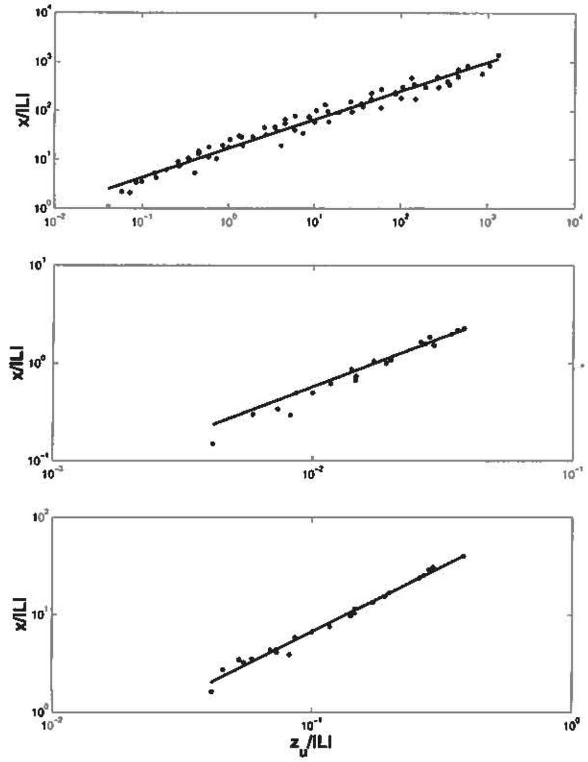

Fig. 1. (a) Scatter plot of $x/|L|$ versus $z_u/|L|$ for unstable conditions (top panel); (b) same as (a) but for neutral conditions (middle panel); (c) same as (a) but for stable conditions (bottom panel).

$$(x/|L|) = \frac{-1}{k^2 \ln(F/S_0)} D(z_u/|L|)^P, \quad (15)$$

where D and P are similarity constants. By applying regression analysis to the results in Fig. 1(a)–(c), we found:

$D = 0.28$; $P = 0.59$	for unstable condition,
$D = 0.97$; $P = 1$	for near neutral and neutral conditions,
$D = 2.44$; $P = 1.33$	for stable condition.

In [13], near neutral conditions was specified for $|z/L| < 0.04$. Here we used a more restricted criterion for near neutral condition with $|z_u/L| < 0.04$ ($\approx |z/L| < 0.02$). (*Note*: The relationship between z_u/L and z/L is not linear.) Rearranging (15), the flux can be estimated by

$$F(x, z_m)/S_0 = \exp\left(\frac{-1}{k^2 x} D z_u^P |L|^{1-P}\right) \quad (16)$$

and the footprint by

$$f(x, z_m) = \frac{1}{k^2 x^2} D z_u^P |L|^{1-P} \exp\left(\frac{-1}{k^2 x} D z_u^P |L|^{1-P}\right). \quad (17)$$

For neutral conditions, the proposed model reduces to Gash's [6] analytical model with $D = 0.97$, which is very close to Gash's analytical value, unity. Upon differentiating $F(x, z_m)/S_0$ with respect to L, we obtain

$$\frac{1}{S_0}\frac{\mathrm{d}F}{\mathrm{d}|L|} = \frac{-1}{k^2 x}(1-P) D z_u^P |L|^{-P} \exp\left(\frac{-1}{k^2 x} D z_u^P |L|^{1-P}\right), \tag{18}$$

which describes the flux sensitivity to a unit change in the Obukhov length. Differentiating (17) with respect to x and setting the resultant equation to zero, we obtain

$$x = \frac{D z_u^P |L|^{1-P}}{2k^2}, \tag{19}$$

which permits explicit estimation of the peak location of the footprint as a function of atmospheric stability and z_u. Eqs. (16)–(19) constitute our proposed model for footprint analysis.

3. Model comparisons

The models of Horst and Weil [8], the Lagrangian stochastic dispersion model [25], and our proposed approach for estimating flux and footprint were contrasted for neutral, unstable, and stable atmospheric conditions. For neutral conditions, Gash's [6] model was also considered. A comparison between the proposed model calculation and field measured water vapor flux evolution with downwind distance over a potato site were then discussed.

3.1. Flux and footprint model comparisons

Fig. 2(a) shows a typical comparison among these models for estimating flux as a function of fetch for unstable condition, where $z_m = 4$ m, $z_0 = 0.04$ m, $L = -50$ m. In Fig. 2(a), good agreement is noted among all these models. Fig. 2(b) is the same as Fig. 2(a), but for footprint estimation. Fig. 2(b) shows that the locations of the peak footprint estimated by all three models are reasonably close. Fig. 3(a) and (b) are the same as Fig. 2(a) and (b), respectively, but for neutral condition. All models compared well with each other in both figures. This agreement also demonstrates that Gash's [6] simple model is very reliable in neutral flows when compared to the more detailed models such as those of Horst and Weil [8]. Fig. 4(a) and (b) shows the same comparisons as Fig. 2(a) and (b), respectively, but for stable condition, where $L = 100$ m. Again, there is good agreement among these models. While footprint peak locations estimated by these models are in close agreement, the magnitudes of the peaks are somewhat different. These differences are neither surprising nor critical since scalar flux magnitude around the peak location changes rapidly within a short distance. In these

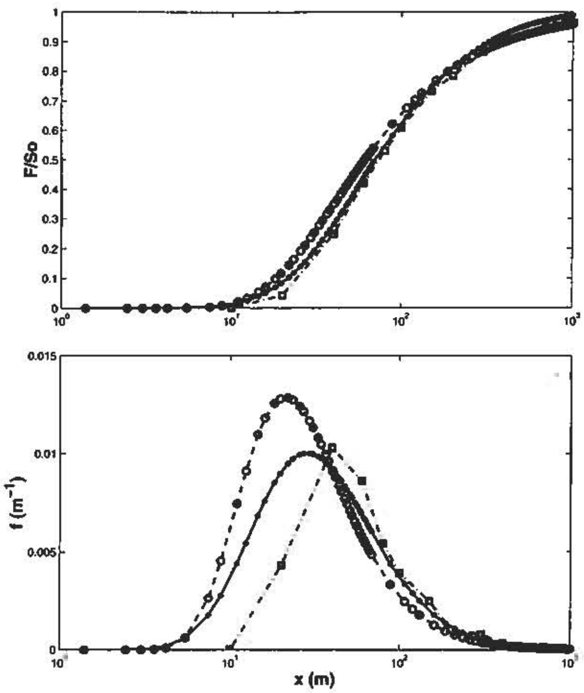

Fig. 2. (a) Comparison between flux estimated by the models of Horst and Weil [8] (closed circles), Thomson [25] Lagrangian stochastic (open squares), and the proposed (open circles) as a function of fetch under unstable condition, where $z_m = 4$ m, $z_0 = 0.04$ m, $L = -50$ m (top panel); (b) same as (a) but for footprint (bottom panel).

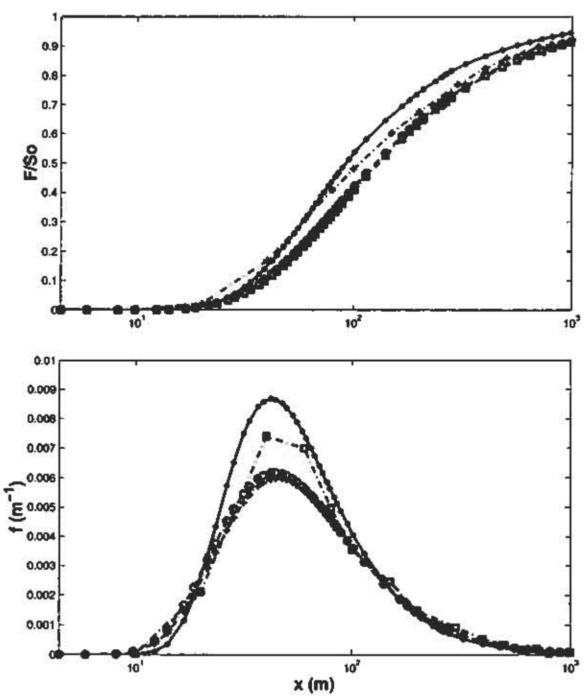

Fig. 3. (a) Same as Fig. 2(a) but for neutral condition. Prediction by Gash's [6] model (plus) is also shown (top panel); (b) same as Fig. 2(b) but for neutral condition. Prediction by Gash's [6] model (plus) is also shown (bottom panel).

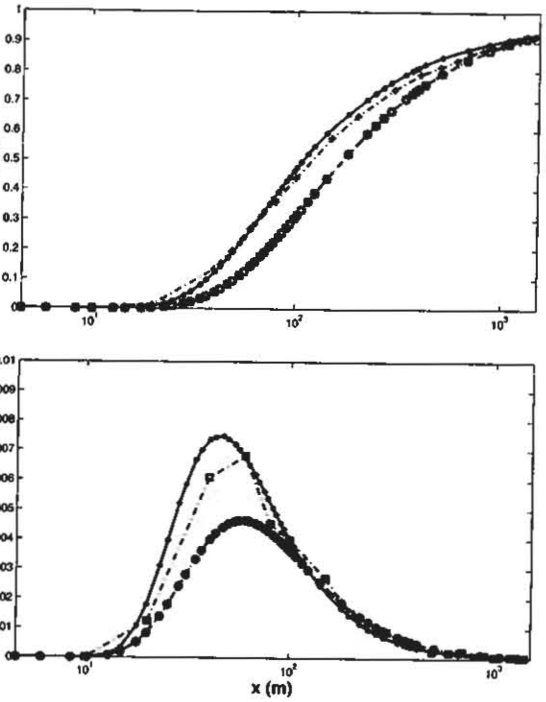

Fig. 4. (a) Same as Fig. 2(a) but for stable condition, where $L = 100$ m (top panel); (b) same as Fig. 2(b) but for stable condition, where $L = 100$ m (bottom panel).

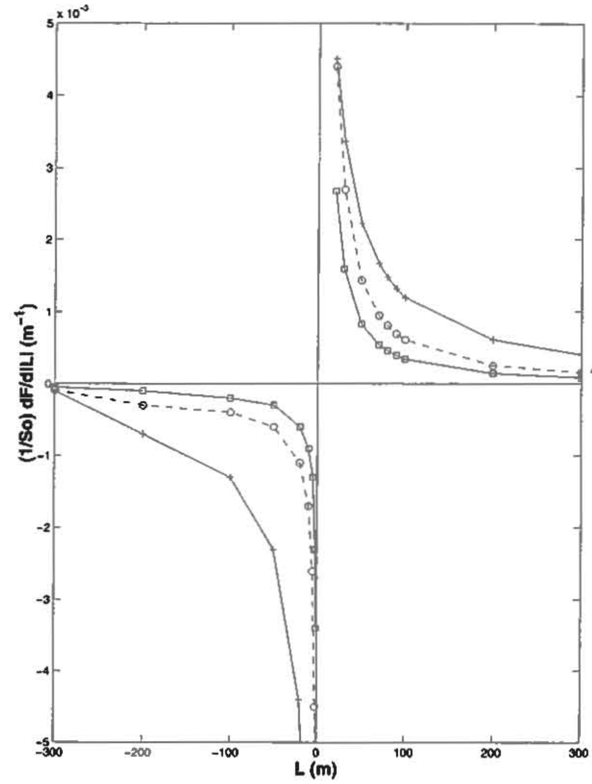

Fig. 5. Flux change, $(1/S_0)(dF/d|L|)$, at different fetches (x) as a function of atmospheric stability, where $z_m = 4.0$ m and $z_0 = 0.04$ m for $x = 100$ m (plus), $x = 500$ m (open circles), and $x = 1000$ m (open squares).

calculations, the height-to-fetch ratios are about 1:100, 1:250, and 1:300 for unstable, neutral, and stable conditions, respectively, as shown in Figs. 2(a), 3(a) and 4(a). These results are consistent with those of Leclerc and Thurtell [16].

Of interest is the fetch-to-height ratio, commonly determined from the 90% constant flux layer, where the flux measurements vary within 10% with height. By setting the value of F/S_0 in (15) to be 0.9 and rearranging the equation, we can express

$$\frac{x}{z_m} = \frac{D}{0.105k^2} z_m^{-1} |L|^{1-P} z_u^P \qquad (20)$$

to calculate such fetch-to-height ratio analytically. From (20), it is obvious that the fetch-to-height ratio changes with measurement height, surface roughness, and stability.

Using (18), Fig. 5 shows how flux changes with a unit change in L at different fetches (x) as a function of atmospheric stability, where $z_m = 4.0$ m and $z_0 = 0.04$ m. Notice that if the measurement is carried out far away from the leading edge, then the measured flux changes less with stability changes.

Of practical importance is the estimation of contributing source area to a specified measurement level. Using (19), Fig. 6 shows the peak location of the footprint for different measurement heights as a function of atmospheric stability, where $z_0 = 0.04$ m. It is obvious that for very unstable conditions, the more co-located the peak location is to the measurement point.

3.2. Field testing

Field testing of these footprint models by single point measurements has been conducted by many investigators [4,12]. Here we test the proposed model with field measurements along a progression of distances downwind a transition from a desert to a potato field. A brief description of this experiment is presented below; details can be found in [1].

The experimental site was an irrigated potato field, which was surrounded by a desert and an *Alfalfa* patch. The crop was irrigated frequently to make sure that the crop did not suffer any water deficit. The surface roughness height was 0.005 m for the desert and 0.05 m for the potato field. A stationary eddy-covariance system set at 800 m downwind from the transition was used to measure turbulent fluxes of momentum, sensible heat, latent heat, and CO_2 above the potato field. A mobile eddy-covariance system was used to measure these fluxes along a progression of distances, 1, 38.4, 72, 91, 136, and 295 m, downwind from the transition. Both systems

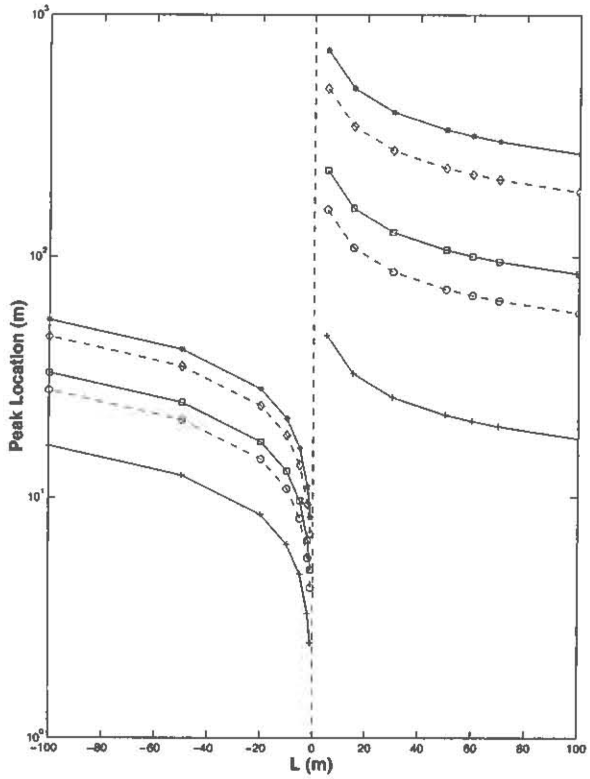

Fig. 6. Peak location of the footprint at different measurement heights (z) as a function of stability, where $z_0 = 0.04$ m for $z = 2$ m (plus), $z = 4$ m (open circles), $z = 5$ m (open squares), $z = 8$ m (diamonds), and $z = 10$ m (stars).

were set at 4 m above the ground and all the data were normalized by the measurements from the stationary system. Field measurements and predictions from a second-order closure model [1] for scalar transport showed that the air was in equilibrium with the potato site at the position of the stationary system. Field measurements also showed that the velocity statistics equilibrated with the potato site over a very short distance from the leading edge. Hence, in a first-order analysis of scalar transport, the velocity statistics are assumed to be planar homogeneous.

At this site, the surface flux upwind is not zero and (2) is not directly applicable. Here the source strength is simply approximated as

$$S(x) = \begin{cases} S_1 & \text{for } x < 0, \\ S_2 & \text{for } x \geqslant 0, \end{cases} \tag{21}$$

where the leading edge is at $x = 0$ and S_1 and S_2 are determined from the measured fluxes at $x = 1$ m and $x = 800$ m, respectively. By superposition, we can calculate the flux using

$$F(x,z) = S_1 \int_{-\infty}^{-x} f(x,z)\,\mathrm{d}x + S_2 \int_0^x f(x,z)\,\mathrm{d}x. \tag{22}$$

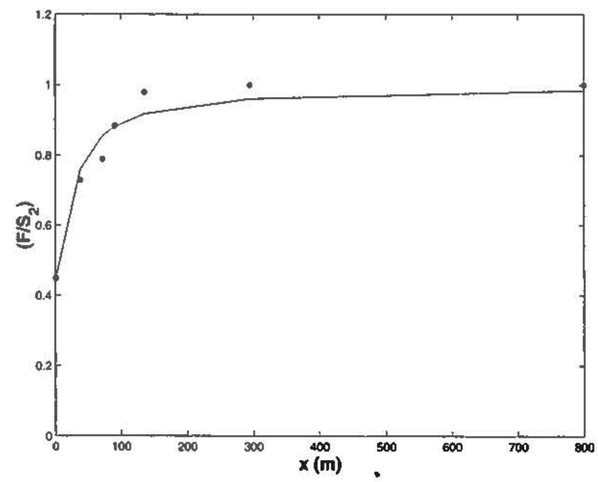

Fig. 7. Variation of the water vapor flux with fetch (downwind distance) above the potato site: the solid line denotes proposed model predictions; the dots denote eddy-correlation measurements.

In (22), for $x \to 0$, $F(x,z) \to S_1$; for $x \to \infty$, $F(x,z) \to S_2$. Using (17), (21) and (22), Fig. 7 shows the variation of water vapor flux with fetch (downwind distance) above the potato site; the solid line denotes the proposed model predictions; the dots denote ensemble-averaged eddy-covariance measurements. In Fig. 7, the predicted water vapor flux variation compared reasonably well with the observation. In this field experiment, the air water vapor concentration increases with distance downwind the transition; thus, the surface water vapor flux will decrease with distance from the interface. Hence, the real source strengths (surface fluxes) along the downwind distance from the discontinuity are stronger than the simple approximation in (21). This perhaps explains why the observed fluxes increase more rapidly downwind of the interface than the model prediction.

4. Conclusion

An approximated analytical model to estimate footprint as a function of atmospheric stability and the length scale z_u was proposed and field-tested. This model is based on dimensional analysis and output results from a Lagrangian stochastic dispersion model. The model performance is comparable to detailed Eulerian and Lagrangian models. Also, good agreement between measured and model predicted water vapor flux evolution with downwind distance over a potato site was demonstrated.

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