Vertical Conductance Formulation

Vertical conductance terms are calculated within the model using data from an input array which incorporates both thickness and vertical hydraulic conductivity in a single term, and using horizontal (or map) areas calculated from cell dimensions. In general, the vertical interval between two nodes, i,j,k and and i,j,k+1, may be considered to contain n geohydrologic layers or units, having vertical hydraulic conductivities $K_1, K_2 \ldots . K_n$ and thicknesses $\Delta z_1, \Delta z_2 \ldots . \Delta z_n$. The map area of the cells around nodes i,j,k and i,j,k+1 is $DELR_j*DELC_i$; the vertical conductance of an individual geohydrologic layer, g, in this area is given by

$$C_g = \frac{K_g \, DELR_j*DELC_i}{\Delta z_g} \qquad (46)$$

The equivalent vertical conductance, $C_{i,j,k+1/2}$, for the full vertical interval between nodes i,j,k and i,j,k+1 is found by treating the n individual geohydrologic layers as conductances in series; this yields

$$\frac{1}{C_{i,j,k+1/2}} = \sum_{g=1}^{n} \frac{1}{C_g} =$$

$$\sum_{g=1}^{n} \frac{1}{\frac{K_g DELR_j*DELC_i}{\Delta z_g}} = \frac{1}{DELR_j*DELC_i} * \sum_{g=1}^{n} \frac{\Delta z_g}{K_g} \qquad (47)$$

rearranging equation (47)

$$\frac{C_{i,j,k+1/2}}{DELR_j*DELC_i} = \frac{1}{\sum_{g=1}^{n} \frac{\Delta z_g}{K_g}} \qquad (48)$$

The quantity $\frac{C_{i,j,k+1/2}}{DELR_j * DELC_i}$ has been termed the "vertical leakance" and is designated $Vcont_{i,j,k+1/2}$ in this report; thus we have

$$Vcont_{i,j,k+1/2} = \frac{1}{\sum_{g=1}^{n} \frac{\Delta z_g}{K_g}} \tag{49}$$

Vcont is the term actually used as input in the model described herein. That is, rather than specifying a total thickness and an equivalent (or harmonic mean) vertical hydraulic conductivity for the interval between node i,j,k and node i,j,k+1, the user specifies the term $Vcont_{i,j,k+1/2}$, which is actually the conductance of the interval divided by the cell area, and as such incorporates both hydraulic conductivity and thickness. The program multiples Vcont by cell area to obtain vertical conductance. The values of Vcont must be calculated or determined externally to the program; this is generally done through an application of equation (49). The Vcont values are actually read as the elements of a two-dimensional input array, $Vcont_{i,j}$, for each layer. Each value of $Vcont_{i,j}$ is the vertical leakance for the interval between cell i,j,k and cell i,j,k+1--that is, for the interval between the layer for which the array is read, and the layer below it. It follows that the Vcont array is not read for the lowermost layer in the model. Although values of Vcont are thus read into the model through a series of two-dimensional input arrays, the discussion in this section will continue to be given in terms of three-dimensional array notation, $Vcont_{i,j,k+1/2}$, to emphasize the fact that the Vcont values refer to the intervals between layers.